**Introduction**

Despite many advances in deep learning, the best lossy compression algorithms are still based on handcrafted algorithms. This is largely due to the non-differentiability of the rate-distortion tradeoff.

The bit-rate as well as the quantization, $\|\cdot\|$, are non-differentiable. We explore a simple approach based on a differentiable approximation of the bit-rate and a redefinition of the derivative of quantization.

**Quantization**

One strategy previously explored is to replace quantization by additive noise [e.g., 1, 2].

$$ [f(x)] \approx f(x) + \varepsilon $$

However, this introduces artefacts which can be perceptually very different from quantization artefacts, and therefore leads to biased distortion estimates.

**Bit-rate estimation**

We express the discrete and non-differentiable $Q$ in terms of a density $q$:

$$ Q(x) = \int_{[-0.5,0.5]^d} q(z+u)/du. $$

We optimize an upper bound on the bit-rate:

$$ -\log_2 Q(x) \leq \int_{[-0.5,0.5]^d} -\log_2 q(z+u)/du. $$

Here, we use Gaussian scale mixtures to model the marginal distribution of coefficients:

$$ \log_2 q(z+u) = \sum_{i,j,k} \log_2 \frac{1}{\alpha} \sum_k n(k) \mathcal{N}(z_{ij}+u_{ij};0,\alpha^2) $$

**Compressive Autoencoder**

A compressive autoencoder consists of an encoder $f$, a decoder $g$, and a probabilistic model $Q$:

$$ f: \mathbb{R}^N \rightarrow \mathbb{R}^M, \quad g: \mathbb{R}^M \rightarrow \mathbb{R}^N, \quad Q: \mathbb{Z}^M \rightarrow [0,1] $$

The probabilistic model is used to assign a number of bits to coefficients produced by the encoder. Our decoder is based on sub-pixel convolutions [1].

Instead of replacing the quantization, we keep it in the forward pass and replace its derivative in the backward pass:

$$ \frac{d}{dy}[y] := \frac{d}{dy}r(y) $$

In practice, we found the identity to work well for $r$.

**Qualitative results**

**PSNR, SSIM, MS-SSIM**

Results on the Kodak dataset:

Mean opinion scores

Following standard practices, we asked 24 naive subjects to rate images on a scale from 1 to 5:

**References**

[1] Shi et al., CVPR, 2016

**Resources**

Compressed images and bit-rates:

http://theis.io/compressive_autoencoder/